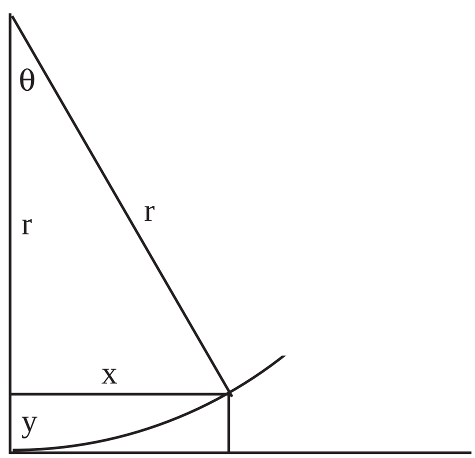
Suppose that a car from a cross street proceeds in a straight line across the path of a bicycle. The bicyclist can try to avoid a collision in one of several ways: stopping, turning, or continuing in a straight line. These three cases can be described more precisely as follows:

1. a bicycle with initial speed v and deceleration a travels forward a distance x.

2. a bicycle with speed v travels foward a distance x at constant speed.

3. a bicycle with speed v travels forward a distance x, but follows a circular path while maintaining a speed v, with the radius of curvature of the path choosen so that the acceleration (perpendicular to the direction of travel) is a.

The travel times for these three cases are t_1, t_2, and t_3 respectively, and the radius of curvature r is given by $\frac{v^2}{a}$. In the following figure,



it is easy to show that $\theta = \arcsin(\frac{x}{r})$ and that $y = r - \sqrt{r^2 - x^2}$. A simple computation then yields

$$t_1 = \frac{v}{a}(1 - \sqrt{1 - \frac{2ax}{v^2}} \quad (1)$$

$$t_2 = \frac{x}{v} \quad (2)$$

$$t_3 = \frac{v}{a}\arcsin(\frac{ax}{v^2}) \quad (3)$$

The interesting range for x is the interval $(0, \frac{v^2}{2a}]$, which corresponds to the distances over which the bicycle can move in a straight line before stopping—if a cyclist can stop before crossing the path of a vehicle, this is obviously the best option. If a collision can be avoided and one can not stop before reaching the car's path, the choices are, to try to pass in front of or behind the car.

Suppose one can pass behind the car, and that Option 1 would result in hitting the car near its rear bumper after traveling forward by a distance x. Swerving to the left may help, but because $t_3 < t_1$, serving will only allow one to avoid the car if the car is going below some speed. This speed is given by

$$v_c = v\left[\frac{1-\sqrt{1-\frac{a^2x^2}{v^4}}}{\frac{v}{a}(1-\sqrt{1-\frac{2ax}{v^2}})-\arcsin(\frac{ax}{v^2})}\right] \qquad (4)$$

and if the car speed is below this value, swerving left will allow one to avoid hitting the car when simply braking would lead to a collision at the rear end of the car.

Finally, suppose one can pass in front of the car, and that option 1 would result in being hit by the front bumper of the car. Both Option 1 and Option 2 will allow one to reach the same point earlier, but there still may not be enough time to pass completely across the path of the car. Suppose Option 2 allows one to cross to the far edge of the car's path before being hit. Now, it turns out that $t_2 < t_3$, so Option 3 will allow one to avoid a collision only if the car cannot cover the distance y in an interval $t_3 - t_2$. This puts a limit on the car's speed of

$$v_c' = \frac{v(1-\sqrt{1-\frac{a^2x^2}{v^4}})}{\arcsin(\frac{ax}{v^2})-\frac{ax}{v^2}} \qquad (5)$$